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Work of a railway sleeper as a structure with variable cross-section - the issue of an equivalent cross-section

Abstract: The article presents an analysis of the work of a sleeper as a construction with variable section, and of the method of determining an equivalent section, constant throughout the length, the utilisation of which would have similar shapes of deflection and bending stress lines in relation to the real, variable cross section. Using an analytical and a numerical model, vertical displacements and stresses for two types of sleepers – PS-94 and PS-08 – were determined. The comparison of the methods allows for calculating an equivalent moment of inertia for analytical calculations, specifically the dynamic ones.

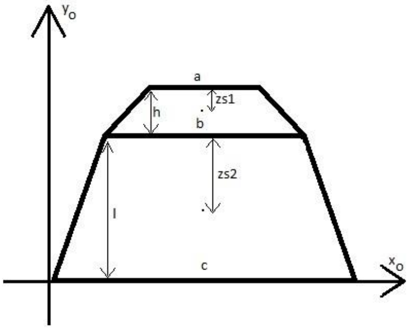
Keywords: Railway surface; Moment of inertia of sleeper; Analytical model; Numerical model

Introduction

In the analysis of dynamic models of the railway surface, usually, foundations are treated as rigid bodies (see [5]). Papers [2.4] are a few examples of the analysis of the variable cross-section of the foundation, limited to static analysis. In [1], it was proposed that - in the case of beams with two clearly distinguishable cross-sections - use the harmonic mean. In analytical dynamic models, the use of a variable cross-section of the undercoat significantly hampers analysis. Therefore, in this work, an attempt was made to determine an equivalent cross-section, i.e. a cross-section with a constant value of inertia moment, the application of which would give similar values of the static response of the underlay (especially displacements) with respect to the real, variable cross-section. The analysis was limited to two types of sleepers: one of the most popular PS-94 sleepers and a heavy foundation PS-08 also operated on the PKP-PLK SA network.

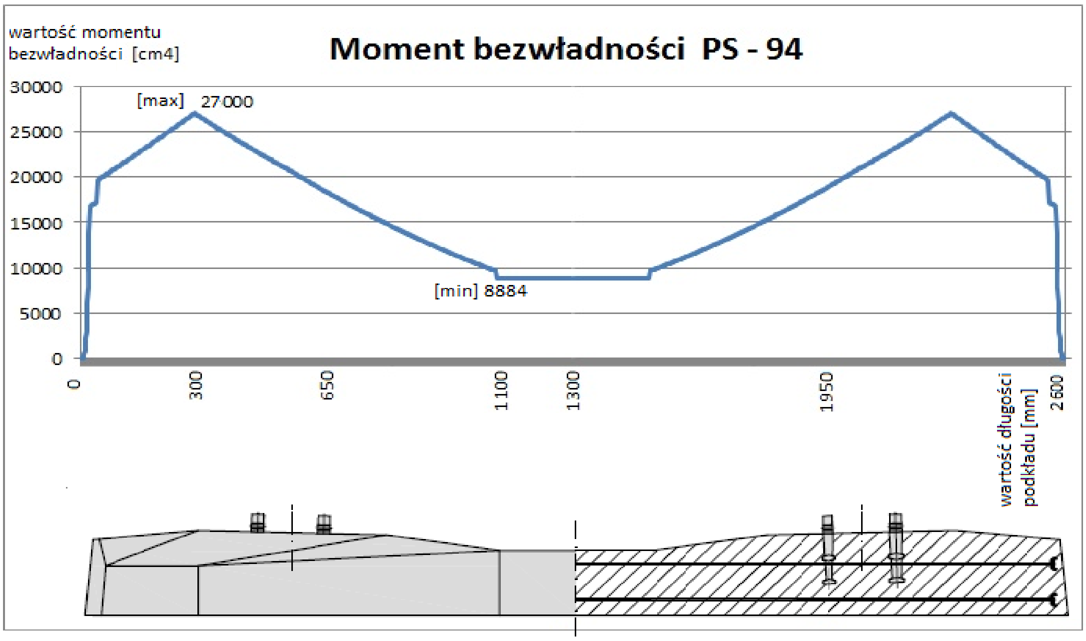
Description of models. Tool for determining the substitute moment of inertia of the railway sleeper.

The development of a physical model of a railway sleeper is a very complex task. It results from the fact that the system formed by individual elements of the track grid substrate, i.e. ballast and soil layer is non-linear and heterogeneous. The work assumes a constant susceptibility to foundation underlayments, described by one elastic parameter. The actual cross-section of the foundation was replaced with two trapezoidal elements. To calculate the substitute moments of inertia for different sections of a prestressed foundation, a diagram was created, shown in the Figure 1.



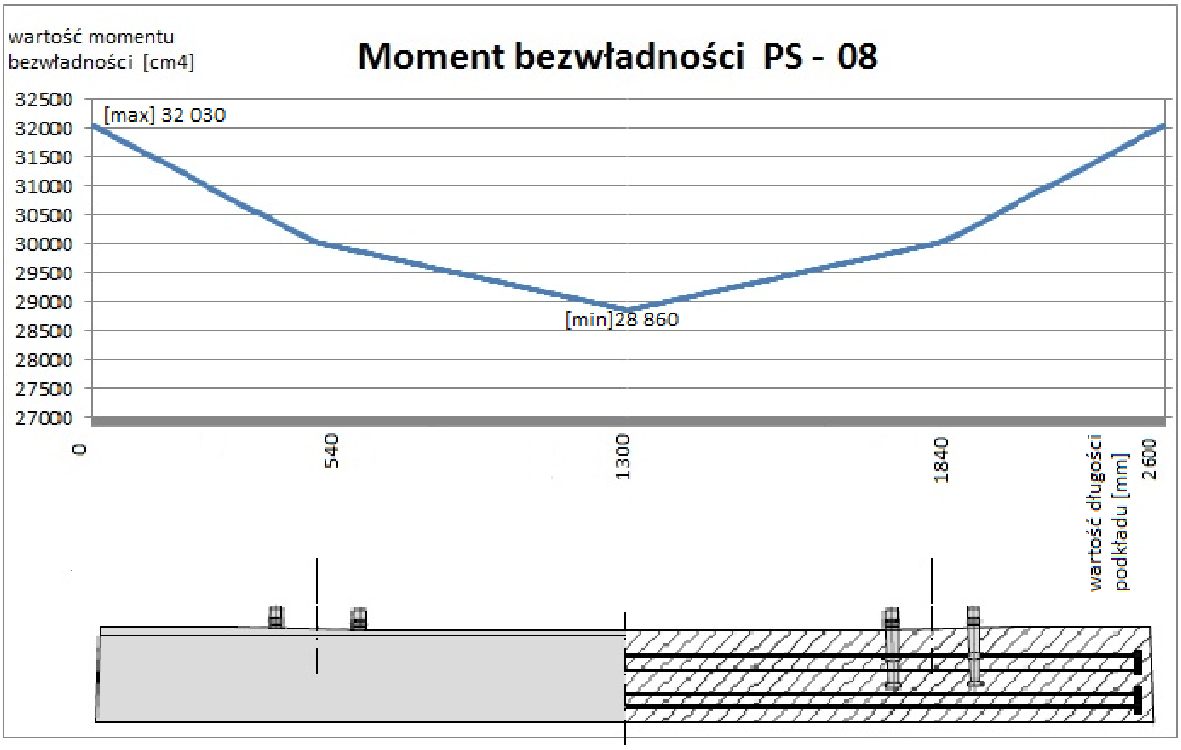
1. The cross-sectional model used to determine the substitute moments of inertia for a prestressed concrete foundation. Own source.

The substitute moment of inertia was calculated using the Steiner's theorem, dividing the cross section into two trapezoids. Using the algorithm created in this way, successive values of substitute moments of inertia were determined. Two primers, PS-94 and PS-08, have been analyzed. The result of the calculations are the following diagrams of substitute moments of inertia. The individual values of the moments of inertia for the PS-94 undercoat shown in the drawing 2.



2. Graph of substitute moments of inertia of PS-94 underlay

Analyzing the above graph, one can observe a large variation in the moment of inertia over the entire length of the underlay. The maximum value is 27,000 cm4 and the minimum is 8,884 cm4. The maximum value is not located at the point of force application (between the anchors of the underlay). On the other hand, the minimum value is in the place most susceptible to cracks, i.e. on the foundation axis. Figure 3 presents the results of the calculation of equivalent moments of inertia for the PS-08 undercoat.



3. Graph of substitute moments of inertia of the PS-08 base

Primer PS-08 is characterized by slight changes in the moment of inertia on the length, the maximum value is 32 030 cm^4, and minimum 28 860 cm^4.

Analytical model

To describe the analytical model, a single-layer surface model in the form of a Bernoulli-Euler beam was adopted. Calculations for this model were made using the Mathcad program: the calculation model was developed at the Department of Railway Transport Infrastructure and Aeronautics of the Cracow University of Technology. The foundation deflection model is described by the differential equation:

$$EI\frac{d^4y}{dx^4} - S\frac{d^2y}{dx^2} + Uy = q(x) \qquad (1)$$

in which the parameters E, S, Ip and U are fixed on the length of the underlay and are indicative:

E – Young's modulus of the primer material [N/m2]
I_p – moment of inertia of the undercoat [m4]
S – the prestressing force in the foundation (- means the tensile force) [N]
U – coefficient of foundation support stiffness [N/m2]
y – vertical displacement of the foundation [m]
q(x) – unit vertical load of the foundation [N]

The foundation was loaded with a system of forces distributed over the length of root pads in the form of a rectangular distribution. To determine the constants of the homogeneous equation present in the solution, zeroing of the second and third derivatives at the ends of the foundation, ie resetting the bending moment M and the lateral force Q at the ends of the foundation was assumed. The solution of a non-homogeneous equation was adopted using the methodology described in [3.5], i.e. developing the deflection line and load in the Fourier series in the interval [0,λ], where λ means the length of the undercoat:

$$q(x) = \frac{a_0}{2} + \sum_{i=1}^{\infty} (a_i \cdot \cos\Omega_i x + b_i \cdot \sin\Omega_i x);$$
$$x \in [0,\lambda]; \Omega_i = \frac{2\pi \cdot i}{\lambda} \tag{2}$$

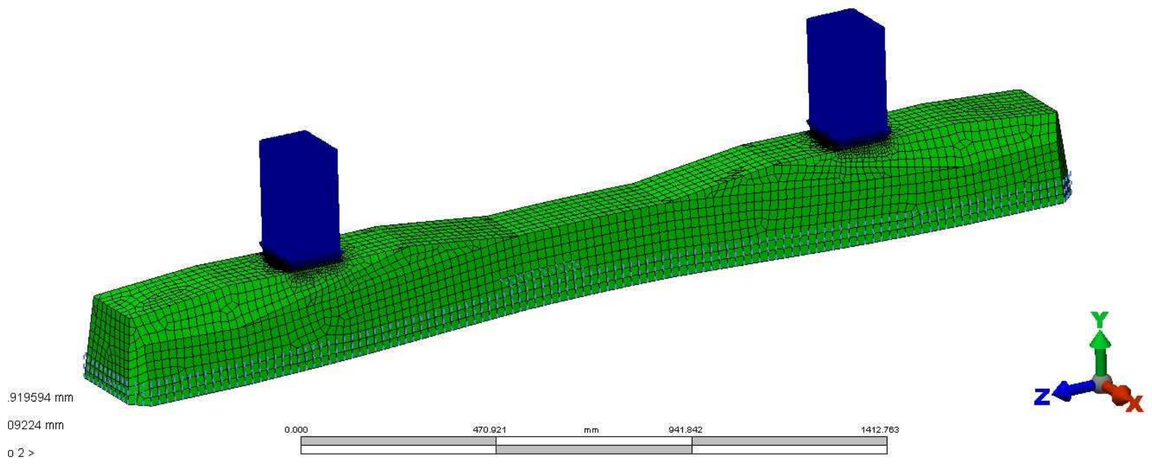
$$y(x) = \frac{y_0}{2} + \sum_{i=1}^{\infty} (A_i \cdot \cos\Omega_i x + B_i \cdot \sin\Omega_i x);$$
$$x \in [0,\lambda]; \Omega_i = \frac{2\pi \cdot i}{\lambda} \tag{3}$$

Constants a_o , a_i and b_i determined based on the development of the known load in the Fourier series based on analytical formulas, and constant y_o, A_i and B_i was determined after differentiation of expressions (3) and substitution of equations (1) with expressions (2) and (3).

Numerical model

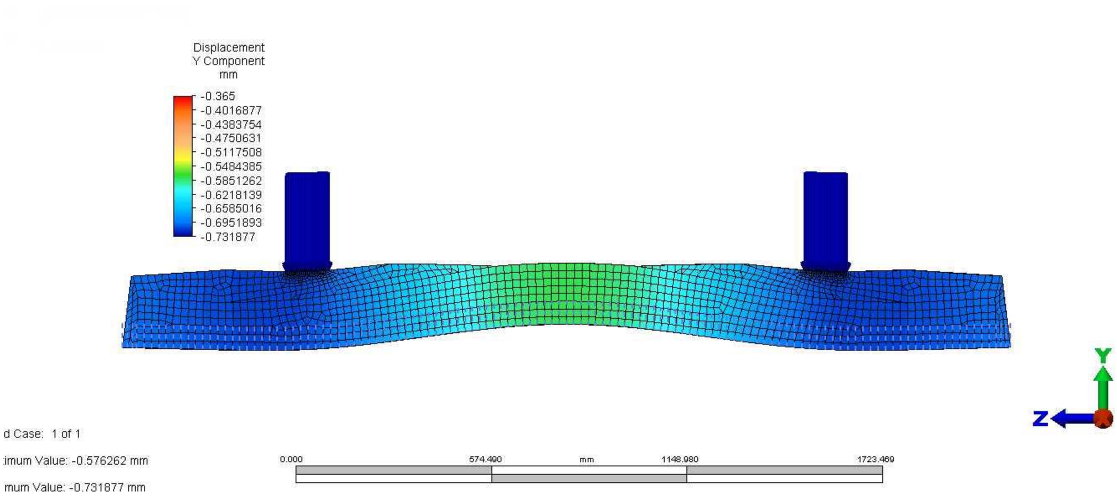
In order to make a numerical model based on the Finite Element Method used to analyze the state of stresses and displacements, three-dimensional blocks of foundations were created. Autodesk Inventor was used for this purpose, where the PS-94 and PS-08 sleeper models were created.

Then in the Autodesk Simulation Multiphysics program, a static analysis of the models was carried out. The previously developed solid has been given material properties corresponding to the elements found in the track. The sub-heap and sub-track have been replaced by string elements with appropriate stiffness. The load from the forces transmitted through the rails has been distributed over the surface corresponding to the field of the rail pad.



4. 3D model of the PS-94 foundation in Autodesk Simulation Multiphysics

The static analysis of the model was carried out for three different values of underlay substrates C_b.

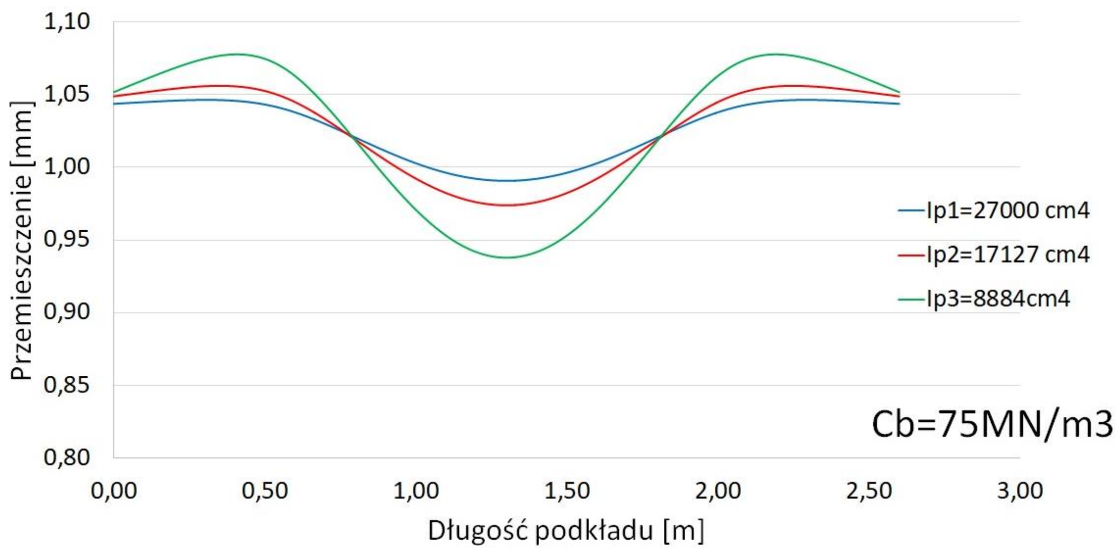


5. 3D model of the PS-94 foundation with a scaled image of deflection

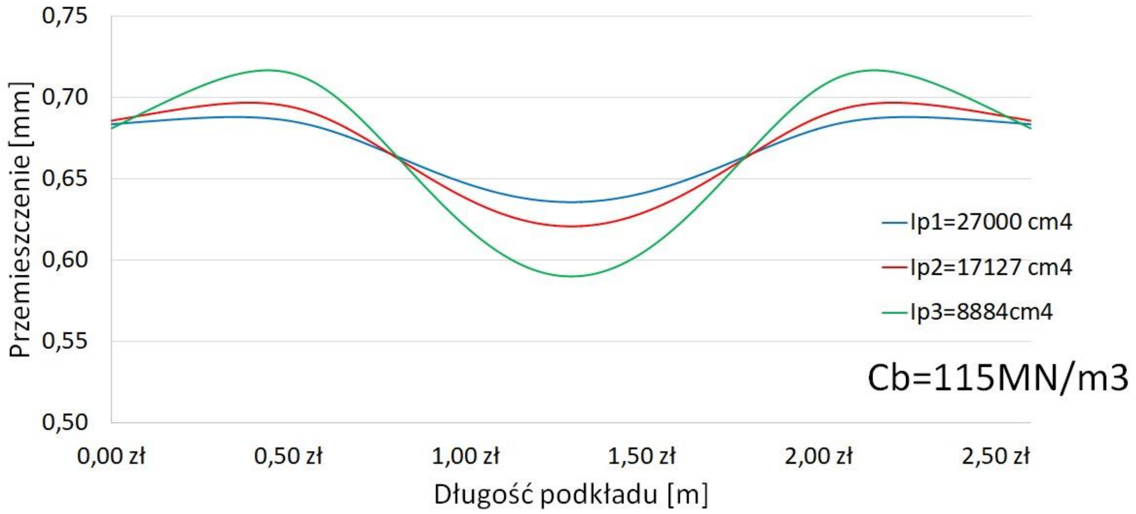
Parametric analysis

Parametric analysis was carried out for two PS-94 and PS-08 pre-stressed concrete sleepers. Using the analytical model as well as the numerical model, the deflection values for three substrate indexes were compared C_b: 75 MN/m3, 115 MN/m3 and 155 MN/m3. In the analytical model three different values of inertia moment of foundation were used for calculation (max, min and average).

Figures 6, 7 and 8 show the PS-94 foundation deflection line for the maximum, average and minimum values of the moment of inertia using an analytical model and various substrate index values.



6. The line of deflection of the PS-94 underlay for the maximum mean and minimum value of the moment of inertia and substrate ground indicator C_b=75 MN/m^3



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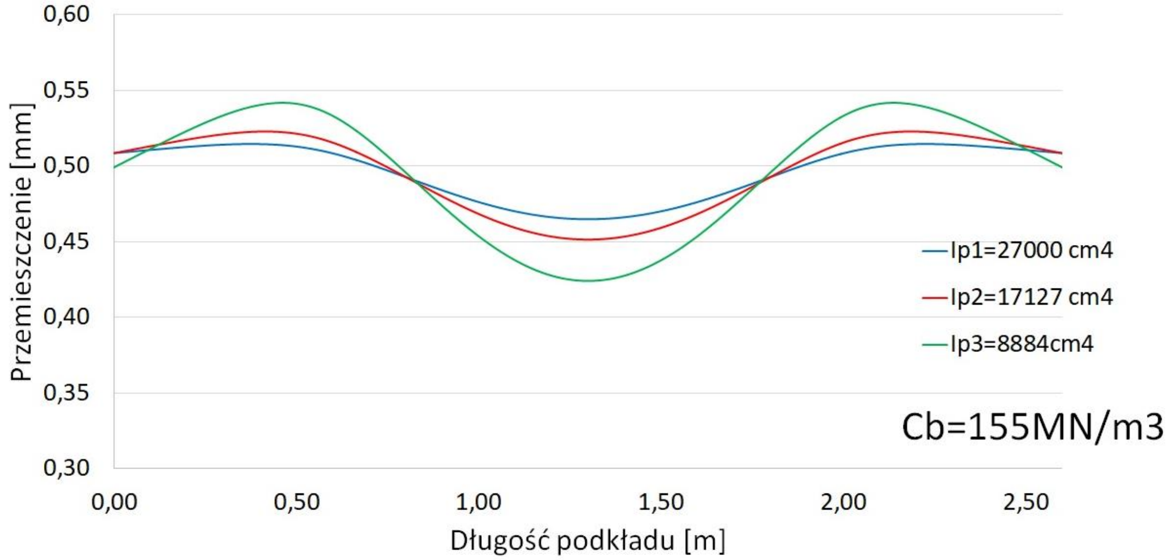
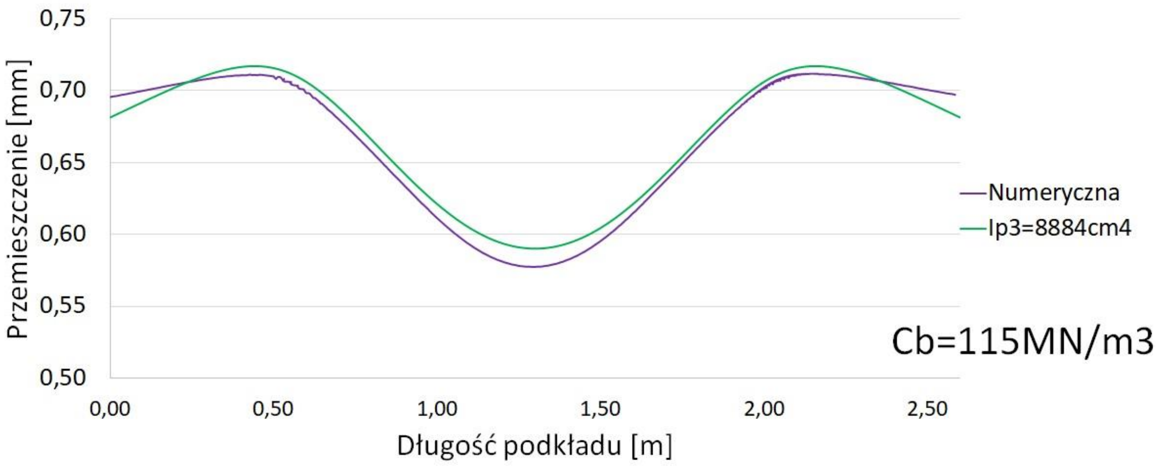
7. The line of deflection of the PS-94 underlay for the maximum mean and minimum value of the moment of inertia and substrate ground indicator C_b=115MN/m^3



8. The line of deflection of the PS-94 underlay for the maximum mean and minimum value of the moment of inertia and substrate ground indicator C_b=155MN/m^3

As a result of the comparative analysis, it was noticed that the deflection line of the PS-94 prestressed concrete foundation is strongly influenced by the assumed moment of inertia. Comparing the results obtained in the analytical analysis with those obtained from the numerical analysis - it turns out that the obtained results from both methods have the most similar values in case of the adoption of the minimum inertia moment of the foundation in the analytical analysis. Figure 9 presents a comparison of the PS-94 foundation deflection line obtained in the numerical method with the results of the analytical analysis for the smallest moment of inertia I_p=8884 cm4. The result for one value of the base substrate coefficient was presented Cb=115MN/m3, the remaining results are shaped similarly.



9. Comparison of the PS-94 backbone deflection line for the numerical and analytical methods including the moment of inertia I_p=8884 cm^4. The value of the ground coefficient C_b=115MN/m^3

The following tables (Tab. 1, 2 and 3) show the minimum and maximum values of foundation displacements for all soil coefficients.

Tab. 1. Comparison of PS-94 backing displacement values for the numerical and analytical methods including the moment of inertia I_p. The value of the ground coefficient C_b=75MN/m^3

Displacement values for primer PS-94 [Base substrate indicator 75MN/m^3]							
	Numerical	**Analytical**					
		różnica [%]	Ip_1=27000 cm^4	różnica [%]	Ip_2=17127 cm^4	difference [%]	Ip_3=8884 cm^4
min	0,9208	**7,05%**	0,9907	**5,30%**	0,9738	**1,85%**	0,9378
max	1,0746	**2,71%**	1,0463	**1,86%**	1,0560	**0,28%**	1,0776

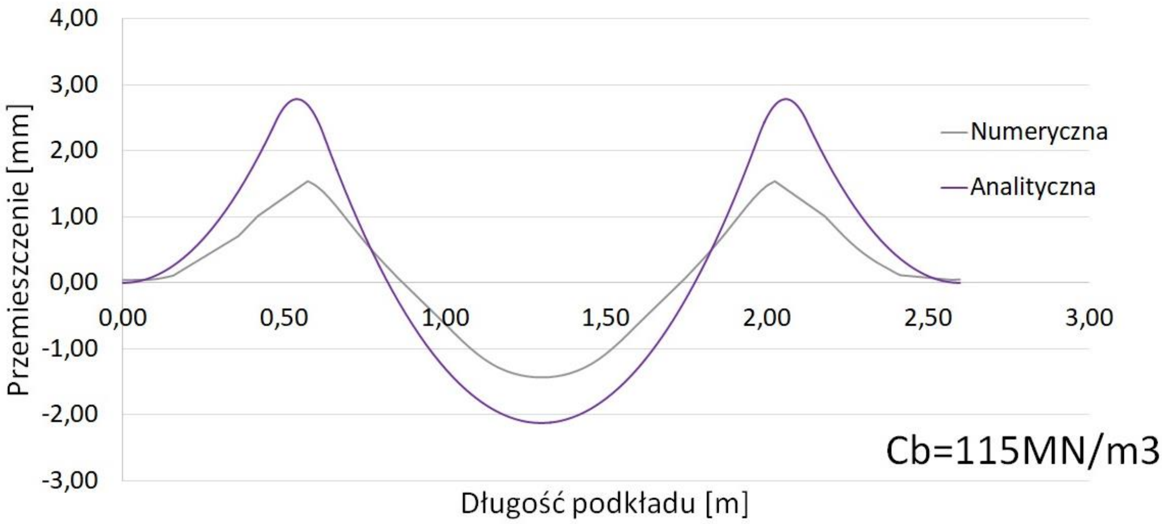
Tab. 2. Comparison of PS-94 backing displacement values for the numerical and analytical methods including the moment of inertia I_p. The value of the ground coefficient C_b=115MN/m^3

Displacement values for primer PS-94 [Base substrate indicator 115MN/m^3]							
	Numerical	**Analytical**					
		różnica [%]	Ip_1=27000 cm^4	różnica [%]	Ip_2=17127 cm^4	difference [%]	Ip_3=8884 cm^4
min	0,5774	**5,85%**	0,6359	**4,36%**	0,6210	**1,28%**	0,5902
max	0,7117	**2,35%**	0,6882	**1,47%**	0,6970	**0,52%**	0,7170

Tab. 3. Comparison of PS-94 backing displacement values for the numerical and analytical methods including the moment of inertia I_p. The value of the ground coefficient C_b=155MN/m^3

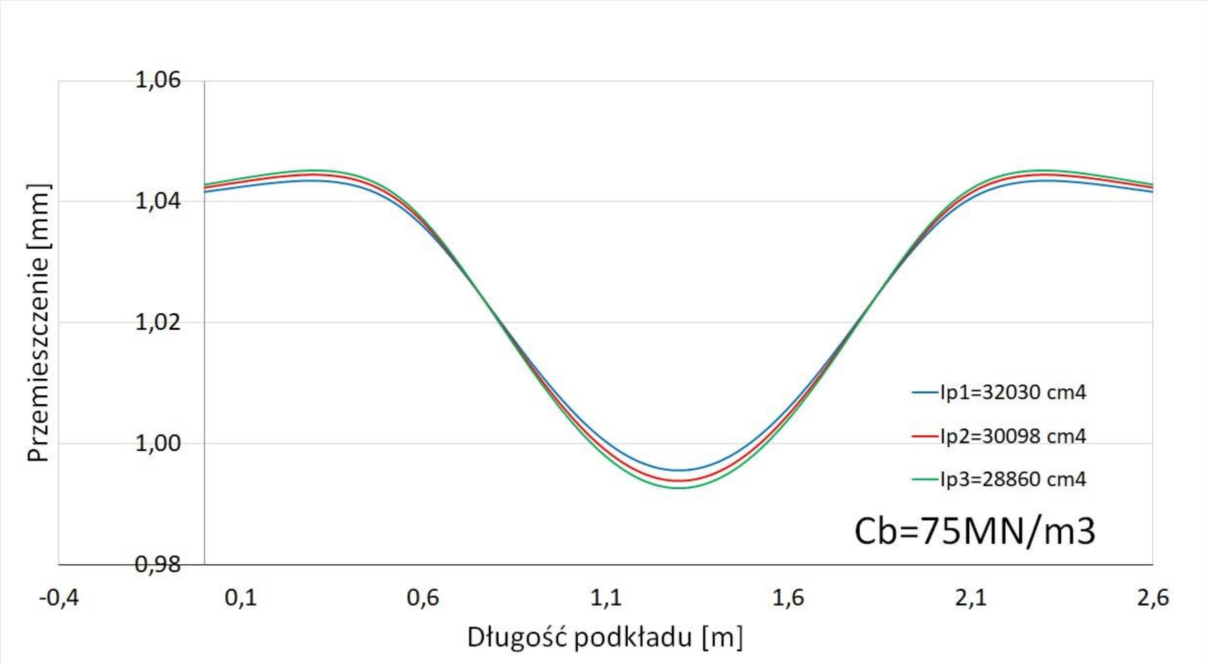
Displacement values for primer PS-94 [Base substrate indicator 155MN/m^3]							
	Numerical	**Analytical**					
		difference [%]	Ip_1=27000 cm^4	difference [%]	Ip_2=17127 cm^4	difference [%]	Ip_3=8884 cm^4
min	0,4332	**3,17%**	0,4649	**1,82%**	0,4515	**0,91%**	0,4241
max	0,5258	**1,11%**	0,5146	**0,29%**	0,5229	**1,61%**	0,5419

For the coefficient Cb=115MN/m^3 it can be noticed that the difference in displacement values calculated in both methods increases with the increase of the moment of inertia value. For the minimum value of the moment of inertia, the values of stresses obtained from the numerical method and the analytical method were also compared. Figure 10 shows the results of this analysis.

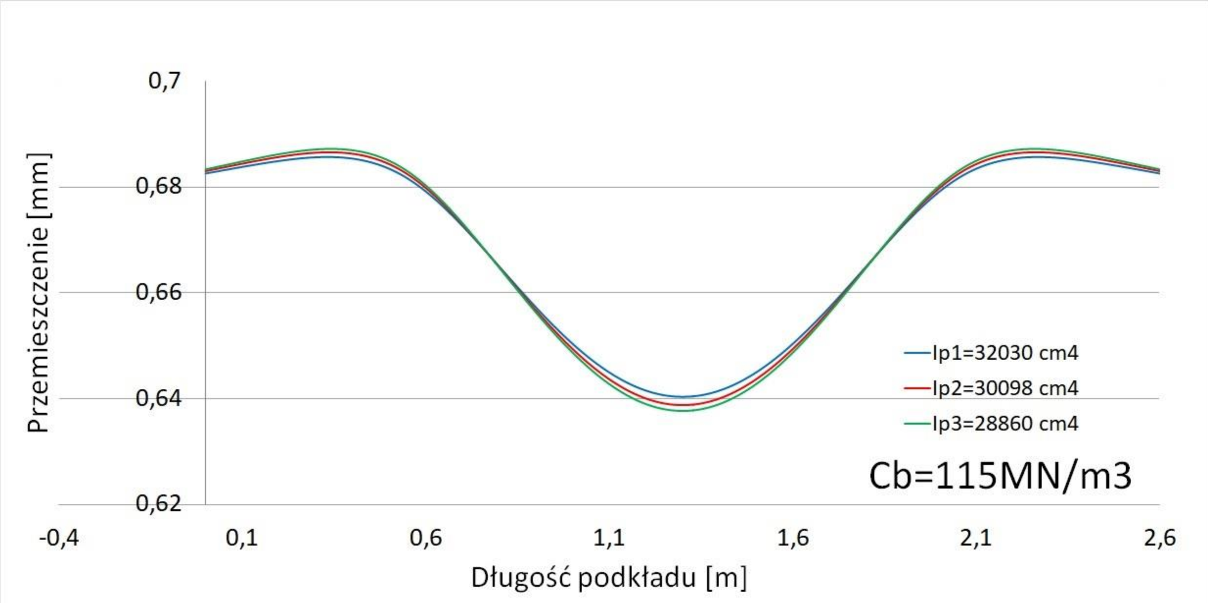


10. Comparison of stress values in PS-94 base for numerical and analytical method including moment of inertia I_p=8884 cm^4. The value of the ground coefficient C_b=115MN/m^3

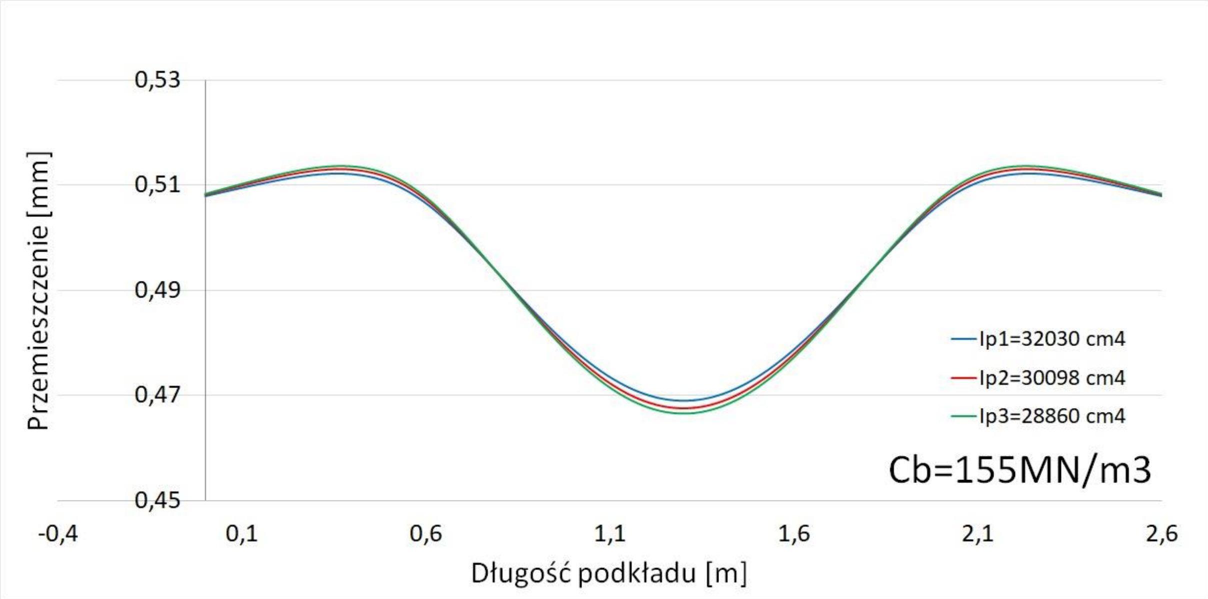
An analogous calculation was also made for a PS-08 pre-tensioned concrete foundation. Figures 11, 12 and 13 show the foundation deflection line PS-08 for the maximum, average and minimum values of the moment of inertia and the different values of the foundation substrate index.



11. The foundation deflection line PS-08 for the maximum mean and minimum value of the moment of inertia and the substrate ground indicator C_b=75 MN/m^3



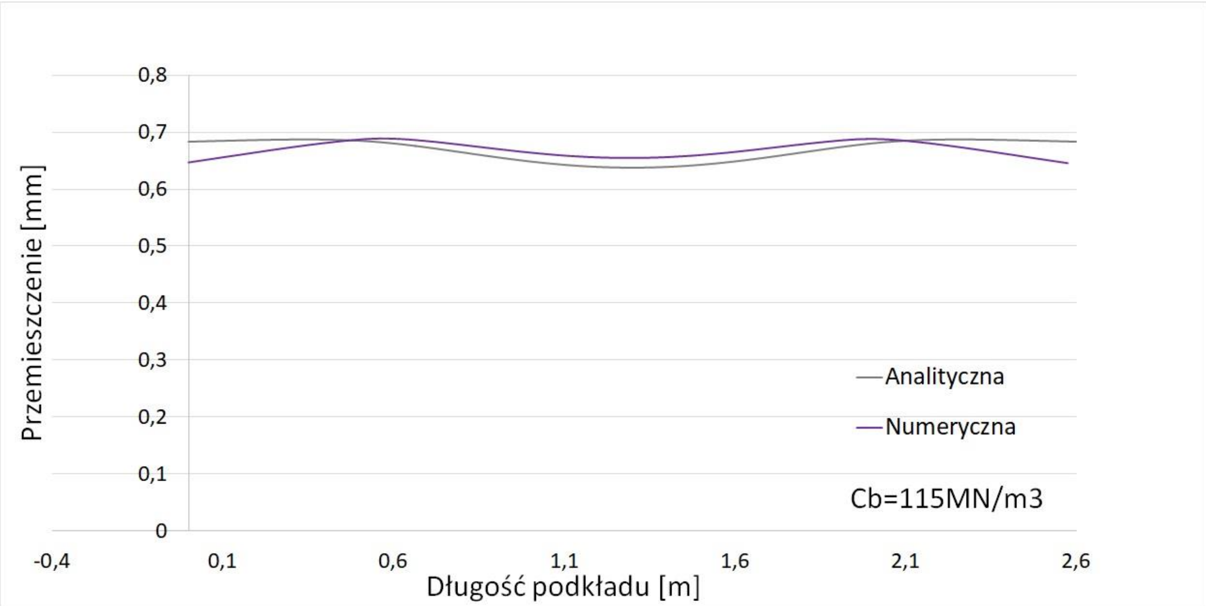
12. The foundation deflection line PS-08 for the maximum mean and minimum value of the moment of inertia and the substrate ground indicator C_b=115 MN/m^3



13. The foundation deflection line PS-08 for the maximum mean and minimum value of the moment of inertia and the substrate ground indicator C_b=155 MN/m^3

After analyzing the obtained results, it was found that the assumed moment of inertia has a much smaller impact on the deflection line of the PS-08 pre-stressed concrete foundation than was the case with the PS-94 base. This dependence results from the fact that the cross-section of the PS-08 underlay is significantly less diversified and the individual moments of inertia (minimum, medium and maximum) differ only slightly in relation to each other..

Figure 14 presents a comparison of the PS-80 foundation deflection line obtained in the numerical method with the results of the analytical analysis for the smallest moment of inertia I_p=8884 cm^4. The result for one value of the base substrate coefficient was presented C_b=115MN/m^3, the remaining results are shaped similarly.



14. Comparison of the PS-08 base deflection line for the numerical and analytical methods including the moment of inertia I_p= 288860 cm^4. The value of the ground coefficient C_b=115MN/m^3

The list of minimum and maximum displacement values of the PS-08 underlay for all ground factors is included in the tables below (**Table 4 to Table 3**).

Tab. 4. Comparison of PS-08 foundation displacement values for the numerical and analytical methods including the moment of inertia I_p. The value of the ground coefficient C_b=75MN/m^3

Displacement values for PS-08 undercoat [Base substrate indicator 75MN/m^3]							
	Numerical	**Analytical**					
		difference [%]	Ip_1=32030 cm^4	difference [%]	Ip_2=30098 cm^4	difference [%]	Ip^3=28860 cm^4
min	0,9910	**0,46%**	0,9956	**0,29%**	0,9939	**0,17%**	0,9927
max	1,0539	**1,00%**	1,0435	**0,94%**	1,0445	**0,83%**	1,0452

Tab. 5. Comparison of PS-08 foundation displacement values for the numerical and analytical methods including the moment of inertia I_p. The value of the ground coefficient C_b=115MN/m^3

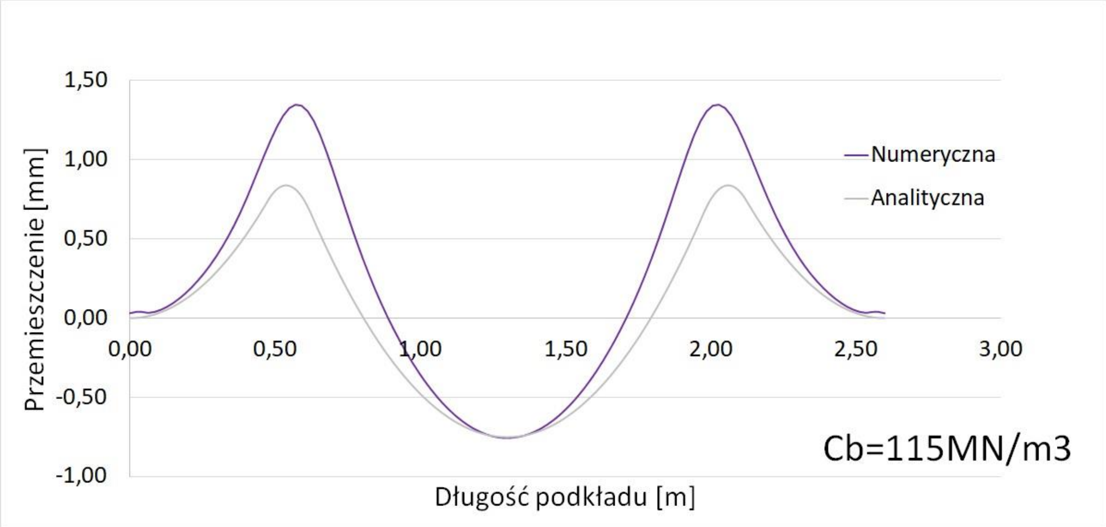
Displacement values for PS-08 undercoat [Base substrate indicator 115MN/m^3]							
	Numerical	**Analytical**					
		difference [%]	Ip_1=32030 cm^4	difference [%]	Ip_2=30098 cm^4	difference [%]	Ip^3=28860 cm^4
min	0,6457	**0,54%**	0,6404	**0,70%**	0,6388	**0,81%**	0,6377
max	0,6889	**0,32%**	0,6857	**0,23%**	0,6866	**0,17%**	0,6872

Tab. 6. Comparison of PS-08 foundation displacement values for the numerical and analytical methods including the moment of inertia I_p. The value of the ground coefficient C_b=155MN/m^3

Displacement values for PS-08 undercoat [Base substrate indicator 155MN/m^3]							
	Numerical	**Analytical**					
		difference [%]	Ip_1=32030 cm^4	difference [%]	Ip_2=30098 cm^4	difference [%]	Ip^3=28860 cm^4
min	0,4631	**0,60%**	0,4690	**0,45%**	0,4676	**0,35%**	0,4666
max	0,5242	**1,20%**	0,5122	**1,12%**	0,5131	**1,06%**	0,5137

The above overview indicates very good compliance of the results obtained (differences on the maximum level of 1.20%. For the coefficient C_b=115MN/m^3 it can be noticed that the displacement values calculated in both methods differ slightly for the smallest value of the moment of inertia - on average about 0,49%.

For the minimum value of the moment of inertia, the values of stresses obtained from the numerical method and the analytical method were also compared. Figure 15 shows the results of this analysis.



15. Comparison of stress values in PS-08 base for numerical and analytical method including moment of inertia I_p=288860 cm^4. The value of the ground coefficient C_b=115MN/m^3

Summary
The adopted, minimum moment of inertia has a much smaller effect on the deflection line of the PS-08 pre-stressed concrete foundation than in the case of PS-94 undercoat. This dependence results from the fact that the cross-section of the PS-08 underlay is significantly less diversified and on the entire length is close to the rectangular one. Nevertheless, the differences obtained with the PS-94 undercoat are not significant - with the average substrate index value of C_b = 115 MN / m3, it is about 3%.
Comparing the results obtained in the analytical analysis with those obtained from the numerical analysis - it turns out that the obtained deflection results from both methods have the most similar values in the case of accepting the minimum inertia moment of the foundation in the analytical analysis. In the case of stresses, there are larger differences - according to the method shown, the value of the inertia memento can be selected to minimize the differences both in the sense of stresses and displacements. It should be noted that the adopted methodology allows for a determination of the equivalent moment of inertia, which is constant on the foundation length, which may be applicable to model analyzes of the railway surface, especially dynamic models.

Source materials
[1] Buczkowski W.: Rozwiązywanie belek o zmiennej sztywności metodą różnic skończonych. Architektura, 8 (3-4), 2009, s. 49-64
[2] Bednarek W.: Wpływ pionowych odkształceń nawierzchni i podtorza na pracę toru bezstykowego. Rozprawy Politechniki Poznańskiej, nr 506, Wydawnictwo Politechniki Poznańskiej, Poznań 2013
[3] Czyczuła W., Kozioł P., Błaszkiewicz D. - On the Equivalence between Static and Dynamic Railway Track Response and on the Euler-Bernoulli and Timoshenko Beams Analogy, Shock and Vibration, Volume 2017 (2017), Article ID 2701715
[4] Kroczak S., Mazur S., Dudek Z.: Teoretyczna analiza wpływu zmiennej sztywności I sposobu podparcia podkładu kolejowego na jego pracę. Materiały VII Konferencji "Drogi Kolejowe", 1993, s. 141-150
[5] Odrowąż M.: Obroty podkładów jako efekt nierównomiernego obciążenia toków szynowych. Praca dyplomowa napisana pod kierunkiem W. Czyczuły, Politechnika Krakowska, Kraków 2016