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DOI: 10.35117/A_ENG_21_06_07_01

Modeling the vehicle-bridge interaction in the conditions of road level deformation

Abstract: The paper presents a method and results of numerical simulations of dynamic interactions between vehicles and the bridge structure, which has defects in the form of excessive permanent deflections of their spans. The simulation analyzes were carried out on the example of a real motorway bridge, on which the presence of such defects was found. A typical 5-axle truck with a total weight of 40 tons was modeled as a moving load. During the analyzes, the main attention was paid to the values of vertical displacements and accelerations of vibrations of the bridge spans generated by heavy vehicles as a function of the intensity of permanent deflections of the structure and the vehicles speed.

Keywords: Dynamic analysis of bridges; Bridge vibrations; Finite Element Method; Structure deformations

Introduction

Permanent deformation of the road grade line on bridge structures is a phenomenon that occurs relatively frequently during the use of the road transport network. Such deformations may arise for various reasons, such as: uncontrolled subsidence of the scaffolding during the construction of the facility, deflections of the structure inconsistent with the design due to rheological phenomena or incorrect prestressing, permanent deflections caused by overloading of the facility as a result of oversize transport or collisions.

Permanent changes in the shape of the bridge structure, most often in the form of deflections of the spans, cause changes in the shape of the road grade line and, as a result, changes in the conditions of vehicle traffic. As a rule, disturbances in the traffic route of vehicles increase the dynamic effects related to the interaction of vehicles and the bridge structure. This may lead to an increase in the value of operational loads, a reduction in the fatigue life of structural elements, as well as a deterioration of the conditions of use of objects due to an increase in the amplitudes of displacements and the value of vibration accelerations of the span structures. This issue is particularly important in relation to bridge structures located along motorways and expressways, where we deal with a large number of heavy vehicles and high traffic speeds.

The phenomenon of the influence of permanent deflections of spans resulting directly in the deformation of the grade line on the operating conditions of the bridge infrastructure is not included in the currently used recommendations for the analysis of existing bridge structures (see e.g. [3, 4, 5, 10]).

Therefore, a numerical simulation analysis of the mutual dynamic interactions of vehicles and the bridge structure with damage in the form of excessive deformation was carried out. The results of the analyzes and the conclusions drawn from them are presented in this article.

Methodology of numerical simulation analyzes

A comprehensive picture of the theoretical, dynamic response of bridge structures to live loads can be obtained using the time analysis of forced vibrations [1, 2, 6, 7, 9]. This requires the formulation of a mathematical model, which is matrix differential equations, the so-called equations of motion, formulated in displacements and its derivatives over time for nodal points of the model. The formulation of the equations of motion, even for systems with several degrees of dynamic freedom, is a demanding task. Analytically, solutions can usually only be obtained for very basic problems. In practice, a necessary step in the dynamic analysis of complex systems is the development and analysis of the numerical FEM model.

In the analysis of the dynamic response of bridge structures to loads and interactions, as a rule, *dynamic implicit analysis* methods are used, in which the values determined in step *i+1* depend both on the value of the size in step *i* and in step *i-1*. These methods are in general (nonlinear problems) iterative methods. The group of implicit integration methods includes the Newmark method [8], which was used in this paper. Its basic theoretical assumptions are briefly discussed below.

In implicit integration methods time is divided into discrete time moments $t_i \in \{t_1, \dots t_n\}$, and then the equation of motion of the model nodes is discretized, i.e. the equation of motion of the dynamical system is transformed to the following form:

$$\mathbf{M\ddot{u}}(t_{i+1}) + \mathbf{C\dot{u}}(t_{i+1}) + \mathbf{Ku}(t_{i+1}) = \mathbf{p}(t_{i+1}) \tag{3}$$

where:

$\mathbf{\ddot{u}}(t_{i+1})$ – vector of generalized vibration accelerations of nodes (in the direction of the degrees of freedom) in the time step i+1,

$\mathbf{\dot{u}}(t_{i+1})$ – vector of generalized vibration velocities of nodes (in the direction of degrees of freedom) in the time step i+1,

$\mathbf{u}(t_{i+1})$ – vector of generalized displacements of nodes (in the direction of degrees of freedom) in the time step i+1,

$\mathbf{p}(t_{i+1})$ – vector of nodal equivalent external loads, i.e. equivalent loads of the system applied at the nodes (in the direction of the degrees of freedom) in the i-th time step.

In the subsequent time steps $i+1$ of the analysis, an approximate solution of the system of equations (1) is sought as a function of the discrete time variable on the basis of the quantities calculated for the previous time t_i, taking into account the initial conditions. The variability of velocity displacements and accelerations of vibrations in particular sub-intervals is approximated $[t_i, t_{i+1}]$, where $t_{i+1} = t_i + \Delta t$.

The length of the integration step Δt is a key parameter influencing the accuracy of the obtained solution. The value of this parameter should be a fraction of the natural period of higher numbers (frequencies). A time step was assumed in the presented analyzes:

$\Delta\mathrm{t} = 0.001$ s, to meet the condition $\Delta\mathrm{t} < \frac{0.1}{\mathrm{f_{max}}}$.

The values of velocity and displacement of vibrations at the end of the time step are calculated on the basis of the values known at the beginning of the time step $\dot{\mathbf{u}}(t_i)$ and $\mathbf{u}(t_i)$ from the formulas:

$$\dot{\mathbf{u}}(t_{i+1}) = \dot{\mathbf{u}}(t_i) + \int_0^{\Delta t} \ddot{\mathbf{u}}(\tau)d\tau \tag{4}$$

$$\mathbf{u}(t_{i+1}) = \mathbf{u}(t_i) + \int_0^{\Delta t} \dot{\mathbf{u}}(\tau)d\tau \tag{5}$$

in which τ is a local time variable in particular time sub-intervals $[t_i, t_{i+1}]$.

Solving equations (4) and (5) requires determining the arbitrary nature of acceleration changes $\ddot{\mathbf{u}}(t_i)$ during a single time step. In the analysis, a constant value of acceleration in the sub-intervals was adopted $[t_i, t_{i+1}]$. This is a so-called *average acceleration method*, in which the changes of vibration accelerations during the time step can be formulated as follows:

$$\ddot{\mathbf{u}}(\tau) = \ddot{\mathbf{u}}(t_i) + \frac{\tau}{\Delta t}[\ddot{\mathbf{u}}(t_{i+1}) - \ddot{\mathbf{u}}(t_i)] \tag{6}$$

In the Newmark method, the following conditions must be met for velocity and displacement at the end of the time interval Δt:

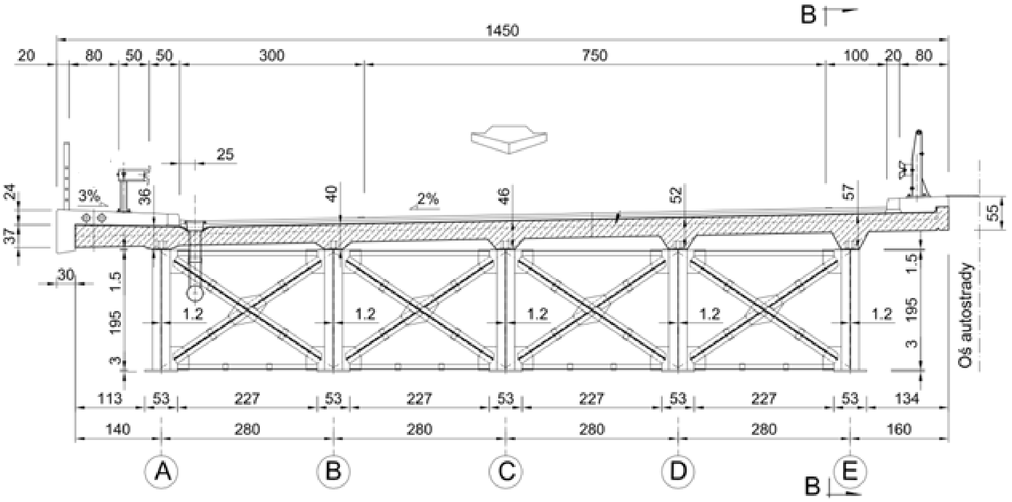
$$\dot{\mathbf{u}}(t_{i+1}) = \dot{\mathbf{u}}(t_i) + \Delta t[(1-\delta)\ddot{\mathbf{u}}(t_i) + \delta\ddot{\mathbf{u}}(t_{i+1})] \tag{7a}$$

$$\mathbf{u}(t_{i+1}) = \mathbf{u}(t_i) + \Delta t\dot{\mathbf{u}}(t_i) + \Delta t^2[(1/2-\beta)\ddot{\mathbf{u}}(t_i) + \beta\ddot{\mathbf{u}}(t_{i+1})] \tag{7b}$$

In the presented analysis, the parameter values were used in the standard Newmark method, i.e. $\delta = 0.5,\ \beta = 0.25(0.5+\delta)^2$.

The bridge structure calculation model

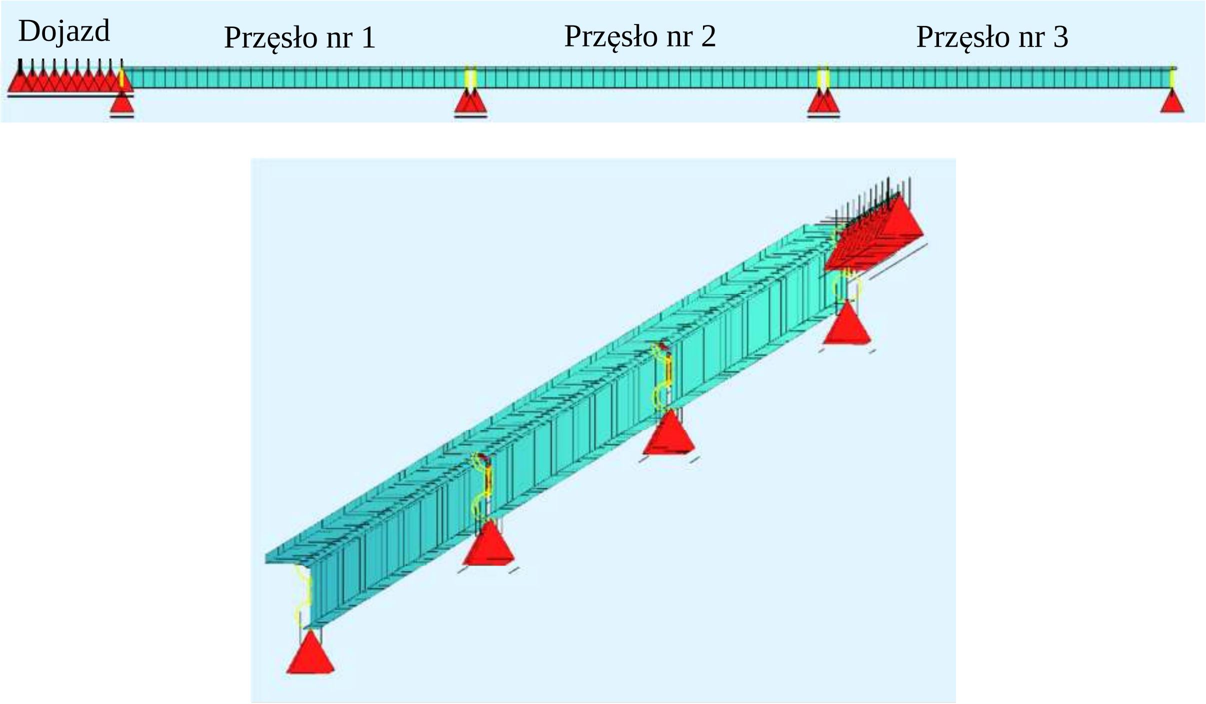
The simulation analyzes were carried out on the example of a real motorway bridge, on which the presence of permanent deformations of the line of one line was found. The object under consideration has two separated multi-span structures, each for one motorway carriageway. The static diagram of the main girders of the bridge is a beam, 7-span structure with the apparent continuity of the structure in support sections, with the use of a thin reinforced concrete slab. The spans are designed as composite steel girders made of 18G2A steel and a reinforced concrete platform slab (Fig. **1**) made of B30 concrete. The composite structure shows excessive, permanent deflections inconsistent with the design, amounting to about 10 cm in the middle of the span of each span (Fig. **2**).



1. Basic dimensions of a single span structure - cross-section

2. Permanent deflection of the main spans

For the purposes of analyzes carried out with the Finite Element Method (FEM), the structure of the bridge spans was mapped in the form of a class model (e1, p2) composed of one-dimensional elements located in a two-dimensional space (Fig. **3**). This model is represented by a single girder separated from three consecutive extreme spans of the analyzed structure.



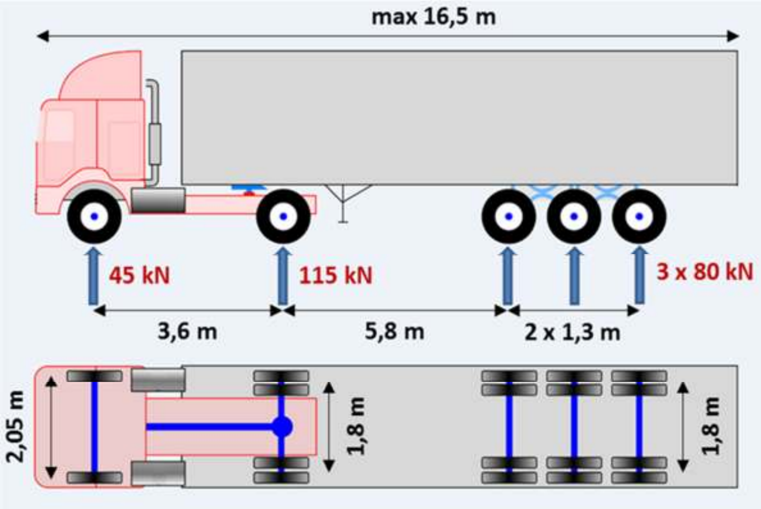
3. Numerical model of the structure developed in the SOFiSTiK program - side view with a division into bays and an axonometric view

For the purposes of dynamic simulation analyzes of the vehicle-bridge structure interaction, comparative analyzes were carried out using two types of models:

- model without damage with perfect geometry of spans - in accordance with the designed grade line,
- a model with damage in the form of permanent deflections of the bridge spans with maximum values up to 150 mm, which were modeled as equivalent geometric imperfections in relation to the original, ideal geometry of the FEM model.

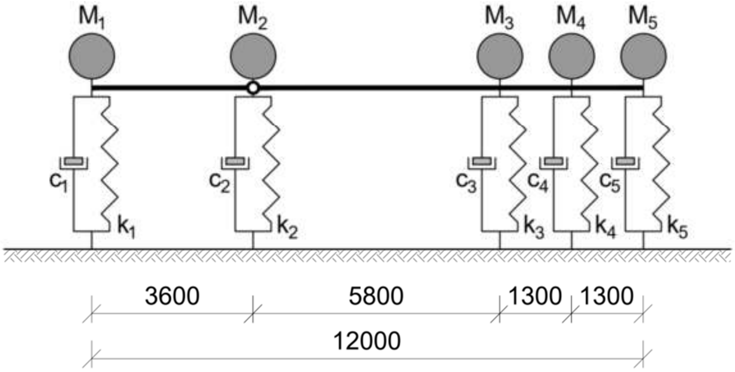
The model of moving loads

The analysis of the work of the structure under load with real vehicle models requires the adoption of not only an effective numerical model of the structure but also an effective representation of a moving vehicle [11, 12]. In the presented simulation analysis, a typical 5-axle truck with a total weight of 40 t was modeled as a moving load (Fig. **4**).



4. Typical 5-axle vehicle weighing 40 tons - wheelbase and axle loads

In the presented analysis, a simplified vehicle model was used, in which the vehicle load was reduced to five masses and five viscoelastic oscillators corresponding to individual suspension axles, connected with each other by a rod of considerable stiffness, with a joint at the place of mass M_2 - see Fig. **5.**



5. Scheme of the suspension modeled as viscoelastic oscillators connected by a rod with a stiffness approaching infinity

In the frequency spectrum of the vehicle impact, the basic natural frequencies of the body vibrations of 2.3 Hz were taken into account, which are the result of the so-called vibrations of the sprung mass. This value is often found in many popular types of heavy road vehicles. The basic parameters of the vehicle model are summarized in Tab. **1**.

The simulation analyzes were carried out for vehicle speeds from 10 m/s (36 km/h) to 30 m/s (108 km/h).

Tab. 1. List of parameters of the numerical vehicle model

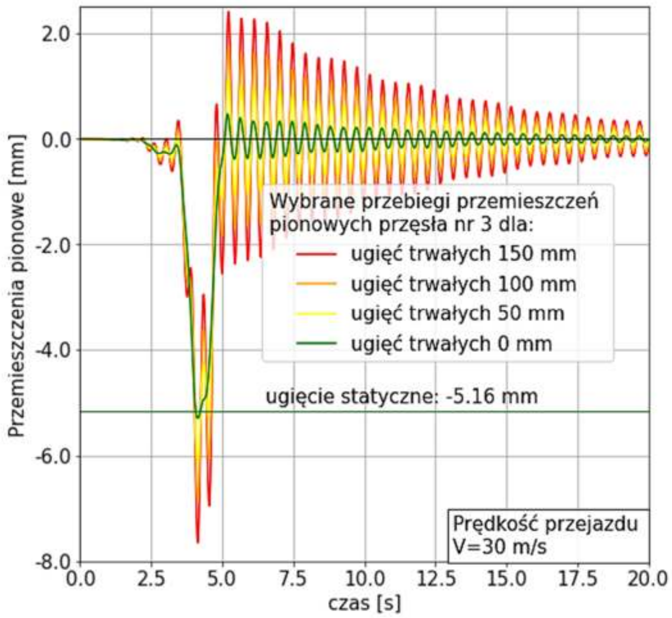
Model parameter	Parameter value
The fundamental frequency of natural vibrations	2,3 Hz
Number of nodal generalized displacements	5
Concentrated masses in the place of the axis	$M_1 = 1.265$ t
	$M_2 = 2.415$ t
	$M_{3-5} = 1.840$ t
Suspension rigidity	$k_1 = 263.9\ \frac{kN}{m}$
	$k_2 = 503.8\ \frac{kN}{m}$
	$k_{3-5} = 383.9\ \frac{kN}{m}$
Suspension damping	$c_1 = 3.654\ \frac{kN}{m \cdot sek}$
	$c_2 = 6.976\ \frac{kN}{m \cdot sek}$
	$c_{3-5} = 5.315\ \frac{kN}{m \cdot sek}$

Parametric analysis results

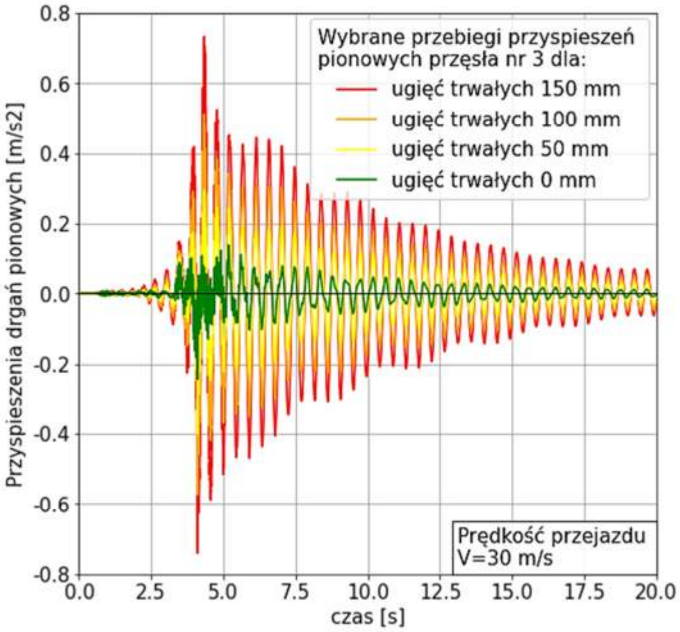
A comprehensive parametric analysis of the dynamic interaction of the adopted 40-ton truck model and the considered type of bridge structure with damage in the form of excessive deformation was carried out. In the subsequent simulations of the vehicle driving over the object, the impact of changing one model identifier (with the values of the remaining parameters set) on the behavior of the structure was investigated.

During the analysis, the main attention was paid to the values of vertical displacements and accelerations of vibrations of the span structures generated by heavy goods vehicles in the main girders as a function of the intensity of permanent deflections of the span structures. The selected results of calculations of changes in physical quantities as a function of time caused by the passage of a 5-axle vehicle weighing 40 tons at a speed of 30 m/s (108 km/h) are shown in Fig. **6** and Fig. **7**:

- Fig. **6** shows changes in the displacement of the cross-section located in the middle of the span of span 3 of the structure model (see Fig. **3**) at different values of permanent deflections of the spans,
- Fig. **7** shows the influence of various values of permanent deflections of the spans on changes in vibration accelerations in the middle of the span of span 3 of the structure model.

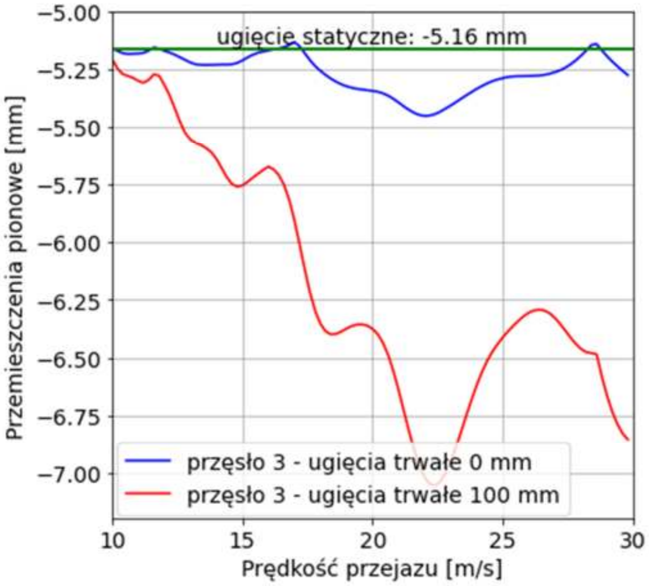


6. Changes in the displacement of the cross-section located in the middle of span 3 caused by the passage of a 5-axle vehicle weighing 40 tons (vehicle speed 30.0 m/s = 108 km/h), with different values of permanent deflections: 0-150 mm

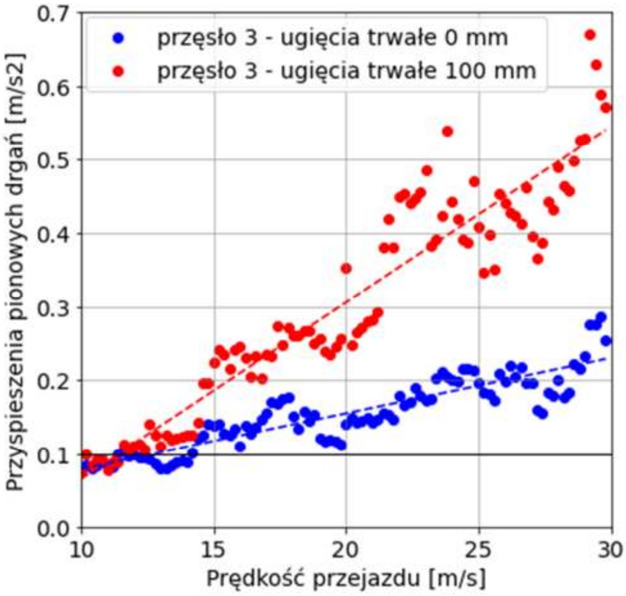


7. Changes in the vibration acceleration of the section located in the middle of span 3 caused by the passage of a 5-axle vehicle weighing 40 tons (vehicle speed 30.0 m/s = 108 km/h), with different values of permanent deflections: 0-150 mm

The impact of the vehicle speed with permanent deformation of the grade line with a maximum value of 100 mm on the maximum values of dynamic vertical displacements is shown in Fig. **8**, while the impact on the maximum vibration acceleration - in Fig. **9**.

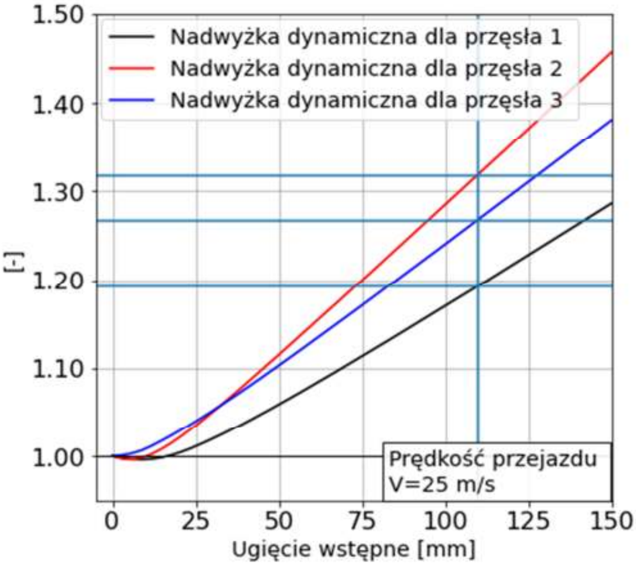


8. The maximum deflections of the section located in the middle of the span of span 3 as a function of the vehicle speed and the value of permanent deflections (0 mm and 100 mm respectively)



9. The maximum vibration acceleration of the section located in the middle of the span of spans 1-3 as a function of the vehicle speed and the value of permanent deflections (0 mm and 100 mm respectively)

The influence of the level of permanent deflections of spans on the increase of the value of the dynamic overload coefficient determined as the ratio of dynamic to static displacements, at a travel speed of 25 m/s, is presented in Fig. **10**.



10. Changes in the value of the dynamic overload coefficient forced by the vehicle passage in sections located in the middle of the span of spans 1-3 as a function of the value of permanent deflections of the spans at a vehicle speed of 25 m/s

Results analysis

Based on the analysis of the results of the conducted numerical simulations, the following conclusions can be drawn:

- The dynamic response of the structure, both in the context of displacements and accelerations of vibrations, is significantly influenced by such factors as the speed of the vehicle passing through the object and the values of permanent deflections of the structure of the spans. The increase in the value of the above-mentioned parameters results in a significant increase in the dynamic effects generated by the vehicle-structure system of the object.
- As a rule, extreme dynamic effects in the structure (in terms of deflections and accelerations of vibrations of the main spans) were obtained in the cross-section located in the middle of span 3 during the passage of vehicle models at a speed of about 23 m/s. For example, for a permanent deflection of spans of 100 mm:
    - the calculated value of the maximum dynamic deflection in span 3 is 30% greater than the maximum static deflection (see Fig. **8**)),
    - the value of the maximum vibration acceleration in span 3 is almost 3 times greater than the maximum vibration acceleration when the same vehicle passes at the same speed, but after the structure with perfect geometry (see Fig. **9**).
- Based on the results of the analysis, it can be concluded that the speed of the considered type of truck, equal to about 23 m/s (82.8 km/h), is the so-called critical speed. The amplitudes of physical quantities that arise in the structure of the spans of the bridge then assume their maximum values.
- The increase in global dynamic effects in the structure generated by vehicle traffic is approximately directly proportional to the value of permanent deflection (Fig. **10**).

Summary

This article shows that the permanent deflections of the spans of bridge structures, resulting in longitudinal unevenness of the road surface, may have a significant impact on increasing the broadly understood dynamic effects generated by the movement of heavy goods vehicles.

These effects (excessive dynamic deflections of the structure, vibration accelerations, and stresses) are generally directly proportional to the value of permanent deflection and have a very negative effect on the durability of the structure (e.g. related to material fatigue, then the ranges of stress variability in the main girders are significantly increased) and user comfort.

In the event of permanent deflections of bridge structures, an essential condition for safe operation is the control and minimization of longitudinal unevenness of the road surface, e.g. by periodically replacing the wearing course of the road surface.

In further stages of research, it is planned to validate the obtained results of the analyzes on the basis of monitoring results, dynamic experimental tests carried out in the conditions of the normal use of the considered bridge, as well as to conduct analogous simulation analyzes using a comprehensive, spatial FEM model taking into account all elements of the bridge spans.

Source materials

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